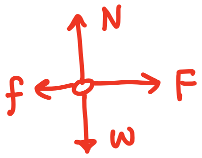
Newton's Laws – Coefficient of Friction

For each problem, include a correctly labeled free-body diagram.

1. A 40 kg box is being pushed by a constant force F across the floor. The coefficient of friction between the floor and the box is $\mu = 0.3$. Find the acceleration for each of the following cases:

a. F = 200 N, horizontally.



$\Sigma F_x = ma$

$F - f = ma$

$\Sigma F_y = 0$

$N - mg = 0$

since $f = \mu N$

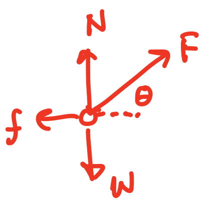
$f = \mu mg$

so $F - \mu mg = ma$

$200 - (.3)(40)(10) = (40)a$

$\boxed{a = 2\ m/s^2}$

b. F = 300 N at an angle of 35° above the horizontal.



$\Sigma F_x = ma$

$F\cos\theta - f = ma$

$\Sigma F_y = 0$

$N + F\sin\theta - mg = 0$

so $f = \mu N = \mu(mg - F\sin\theta)$

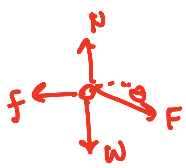
so $F\cos\theta - \mu(mg - F\sin\theta) = ma$

$(300)\cos 35 - (.3)[(40)(10) - (300)\sin 35] = (40)a$

$\boxed{a = 4.4\ m/s^2}$

c. F = 300 N at an angle of 20° below the horizontal.

Hey! It's the same as part b – just change the angle!



So $\theta = -20°$

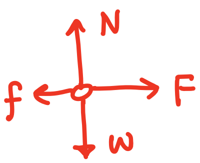
$F\cos\theta - \mu(mg - F\sin\theta) = ma$

$(300)\cos(-20) - (.3)[(40)(10) - 300\sin(-20)] = 40a$

$\boxed{a = 3.3\ m/s^2}$

d. F = 100 N, horizontally. *(Be careful!)*

Looks just like part a!



So $F - \mu mg = ma$

$100 - (.3)(40)(10) = (40)a$

$100 - 120 = 40a$

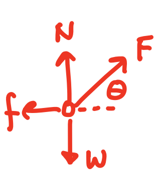
$-20 = 40a$

$a = -\frac{1}{2}\ m/s^2$ HUH??*

That makes NO sense! wait a sec – go back to the green highlights. f only need 100 N to keep thing @ rest – so $\boxed{a = 0\ m/s^2}$

2. A 15 kg box is being pulled by a force F at an angle of 30° above the horizontal. If the coefficient of friction between the box and the floor is $\mu = 0.4$, what is the maximum F can be and not accelerate the box?

* If object was moving, that answer would be fine.



$\Sigma F_x = ma$

$F\cos\theta - f = 0$

$\Sigma F_y = 0$

$N + F\sin\theta - mg = 0$

So $N = mg - F\sin\theta$

& $f = \mu(mg - F\sin\theta)$

So $F\cos\theta = f$

$F\cos\theta = \mu(mg - F\sin\theta)$

$F\cos\theta = \mu mg - \mu F\sin\theta$

$F(\cos\theta + \mu\sin\theta) = \mu mg$

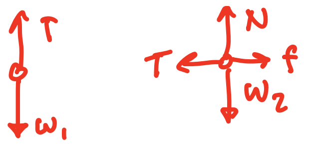
$F = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta} = \dfrac{(.4)(15)(10)}{\cos 30 + (.4)\sin 30}$

$\boxed{F = 56.3\ N}$

Newton's Laws – Coefficient of Friction

3. A mass M is resting on horizontal table and is attached by a string to a mass m that is hanging from a pulley. If the coefficient of friction between M and the table is μ, what is the maximum that m can be and not accelerate M?

on m: $\Sigma F = 0 \rightarrow T - mg = 0 \rightarrow T = mg$



$w_1 = mg$ $w_2 = Mg$

on M: $\Sigma F_x = Ma \rightarrow T - f = 0$

$\Sigma F_y = 0 \rightarrow N - Mg = 0$

Since $f = \mu N$

$T - f = 0$

$T - \mu Mg = 0$

so $mg - \mu Mg = 0$

$\boxed{m = \mu M}$

4. A 250 gram mass is sliding with constant speed down an inclined plane with a base angle of 20°. What is the coefficient of friction between the mass and the inclined plane?

$m = .25\ kg$
$\theta = 20°$

constant speed so $\Sigma F = 0$!

$\Sigma F_{\parallel} = 0$ $\Sigma F_{\perp} = 0$

$mg\sin\theta - f = 0$ $N - mg\cos\theta = 0$

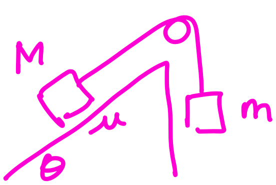
$f = mg\sin\theta$ $N = mg\cos\theta$

Since $f = \mu N$

$mg\sin\theta = \mu mg\cos\theta$

$\mu = \tan\theta = \tan 20$ $\boxed{\mu = 0.36}$

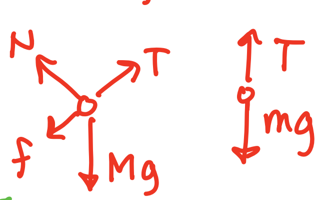
5. A 0.5 kg mass is on an inclined plane with base angle of 30°. The coefficient of friction between the mass and the plane is 0.35. The 0.5 kg mass is attached by a string to a little mass m that is hanging from a pulley from the top of the ramp. If the system is to remain at rest, what are the minimum and maximum that m can be?



↳ if m too big, system accelerates clockwise (m goes down)
if m too small, system accelerates counter-clockwise (m goes up.)

$M = 0.5\ kg$
$m = ?$
$\theta = 30°$
$\mu_s = 0.35$
Static b/c @ rest.

Case 1, Max m)



friction trying to prevent block from sliding up the hill

$\Sigma F_{\parallel} = 0 \rightarrow T - f - Mg\sin\theta = 0$

$\Sigma F_{\perp} = 0 \rightarrow N - Mg\cos\theta = 0$

$\Sigma F = 0 \rightarrow T - mg = 0$

So $T = mg$ & $N = Mg\cos\theta$ & $f = \mu N$

$mg - \mu Mg\cos\theta - Mg\sin\theta = 0$

$m = \mu M\cos\theta + M\sin\theta$

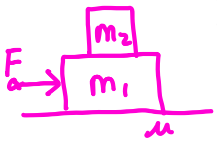
FINISHED @ END

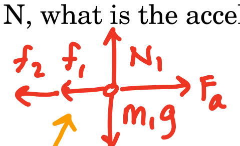
So $m = (.35)(.5)(\cos 30) + (.5)(\sin 30)$

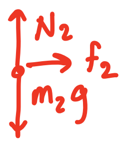
$\boxed{m = 0.40\ kg}$

Newton's Laws – Coefficient of Friction

6. A 3 kg box is resting on top of a 5 kg box, which is on a horizontal table. The coefficient of friction between the box and the table is 0.3. If the bottom box is pushed by a horizontal force of 40 N, what is the acceleration of the boxes, assuming the little box stays on top of the big box.







$m_2 = 3$ kg
$m_1 = 5$ kg
$\mu = 0.3$
$F_a = 40$ N

$f_1 = \mu N_1$

f_2 is friction between boxes
f_1 is friction between table and bottom box

System: $\Sigma F_{sys} = m_{sys} a$

$F_a - f_1 = (m_1 + m_2) a$ — f_2 is internal

on m_2: $\Sigma F_y = 0 \rightarrow N_2 - m_2 g = 0 \rightarrow N_2 = m_2 g$

on m_1: $\Sigma F_y = 0 \rightarrow N_1 - m_1 g - N_2 = 0 \rightarrow N_1 = (m_1 + m_2) g$

$f_1 = \mu N_1 = \mu (m_1 + m_2) g$

Finally: $F_a - \mu (m_1 + m_2) g = (m_1 + m_2) a$

$40 - (.3)(5+3)(10) = (5+3) a$

$40 - 24 = 8a$

$\boxed{a = 2\ m/s^2}$

7. In the previous problem, what must be the minimum coefficient of friction between the two boxes so that the little box stays on top of the big box?

on m_2: $\Sigma F_x = m_2 a \rightarrow f_2 = m_2 a \rightarrow \mu_2 N_2 = m_2 a$

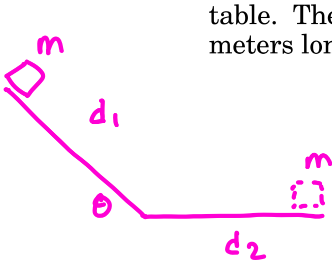
$\Sigma F_y = 0 \rightarrow N_2 - m_2 g = 0$

$N_2 = m_2 g$

$\mu_2 m_2 g = m_2 a$

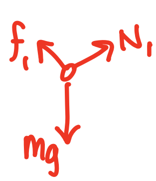
$\mu_2 = \frac{a}{g} = \frac{2}{10} = \boxed{0.2}$

8. A box slides from rest down an inclined plane with base angle 40° and then onto a flat horizontal table. The coefficient of friction between the box and both surfaces is 0.2. If the ramp is 1.5 meters long, how far on the table does the box slide before coming to rest?



$\theta = 40°$
$\mu = 0.2$
$d_1 = 1.5$ m
$v_i = 0$ m/s
$v_f = 0$ m/s
$d_2 = ?$

on incline:



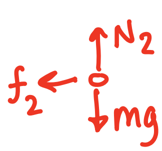
$\Sigma F_\perp = 0 \rightarrow N_1 - mg\cos\theta = 0 \rightarrow N_1 = mg\cos\theta$ on incline

$\Sigma F_\parallel = ma_1$ — $mg\sin\theta - f_1 = ma_1$ & $f_1 = \mu N_1$

$mg\sin\theta - \mu mg\cos\theta = ma_1$

$a_1 = g\sin\theta - \mu g\cos\theta$ — a_1 is to the right

on table:



$\Sigma F_y = 0 \rightarrow N_2 - mg = 0 \rightarrow N_2 = mg$

$\Sigma F_x = ma_2$ — $f_2 = ma_2$ — $\mu mg = ma_2$ & $f_2 = \mu N_2$

Note, a_2 is to the left — $a_2 = \mu g$

Finally, calling the max speed "v"

$v_f^2 = v_i^2 + 2a\Delta x$

incline: $v^2 = 0^2 + 2a_1 d_1 \rightarrow v^2 = 2a_1 d_1 \rightarrow$ So $2a_1 d_1 = 2a_2 d_2$

table: $0 = v^2 - 2a_2 d_2$ $\quad v^2 = 2a_2 d_2$

Newton's Laws – Coefficient of Friction

Answers:

1 a) $2\ m/s^2$ b) $4.4\ m/s^2$ c) $3.3\ m/s^2$ d) $0\ m/s^2$

2) 56.3 N

3) μM

4) 0.36

5) 0.098 & 0.4 kg

6) $2\ m/s^2$

7) 0.2

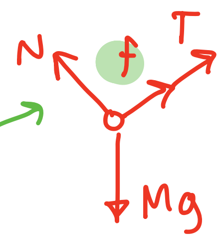
8) 3.65 m

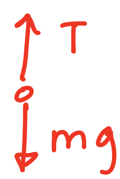
$$d_2 = \frac{a_1 d_1}{a_2} = \frac{(g\sin\theta - \mu g\cos\theta)\,d_1}{\mu g}$$

$$d_2 = \frac{[10\sin 40 - (.2)(10)(\cos 40)](1.5)}{(.2)(10)}$$

$$\boxed{d_2 = 3.67\ m}$$

5 continued) Case 2, minimum m.





because friction trying to hold up block and prevent it from sliding down the ramp.

Notice that the only difference is the direction of friction.

$\Sigma F_{\parallel} = 0 \rightarrow T + f - Mg\sin\theta = 0$

$\Sigma F_{\perp} = 0 \rightarrow N - Mg\cos\theta = 0$

$\Sigma F = 0 \rightarrow T - mg = 0$

So $T = mg$ & $N = Mg\cos\theta$ & $f = \mu N$

$$mg + \mu Mg\cos\theta - Mg\sin\theta = 0$$

$$m = M\sin\theta - \mu M\cos\theta$$

$$= (.5)\sin 30 - (.35)(.5)\cos 30$$

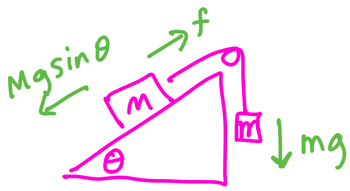
$$\boxed{m = 0.098\ kg}$$

ALTERNATE SOLUTION

Treat this as a system

$$\Sigma F_{sys} = m_{sys}\,a$$

In this case, it is harder to draw a nice free body diagram, but we basically <u>ignore</u> the internal forces. In this case, that means ignoring the tension in the string. Some forces cancel out – the normal force & $w_\perp$ on the mass on the incline. That leaves 3 forces - 2 trying to rotate system clockwise (f & mg) and 1 trying to rotate the system counter clockwise ($Mg_{\parallel}$)



So $\Sigma F_{sys} = m_{sys} a$

$$Mg\sin\theta - f - mg = 0$$

$$Mg\sin\theta - \mu Mg\cos\theta - mg = 0$$

$$\boxed{m = M\sin\theta - \mu M\cos\theta}$$

Still need the $f = \mu N$ & $N = Mg\cos\theta$ from before!

which is what we got earlier